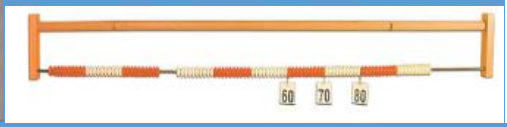
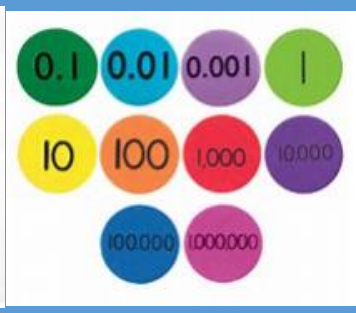
Treloweth School

Agreed Approach to Calculation Document

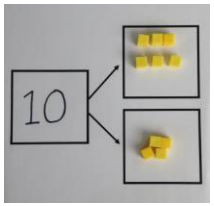
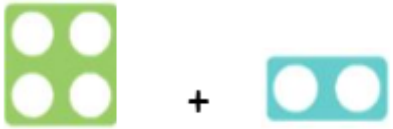
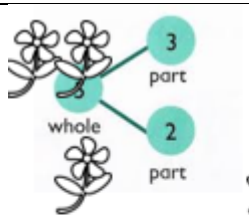
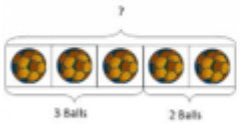
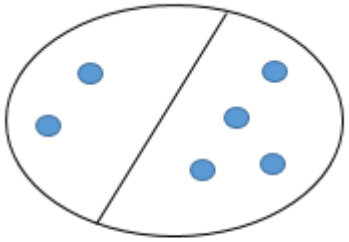
Core manipulatives to support depth of learning from EYFS to Y6:



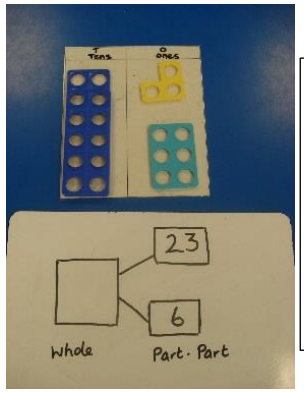
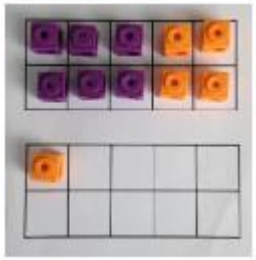
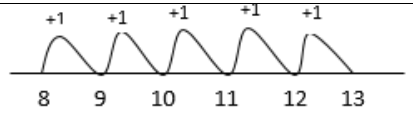
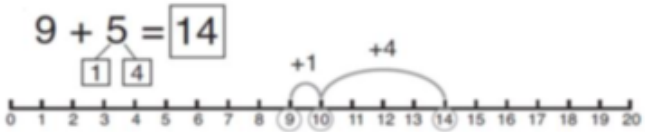
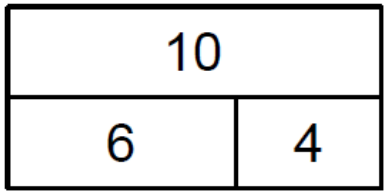
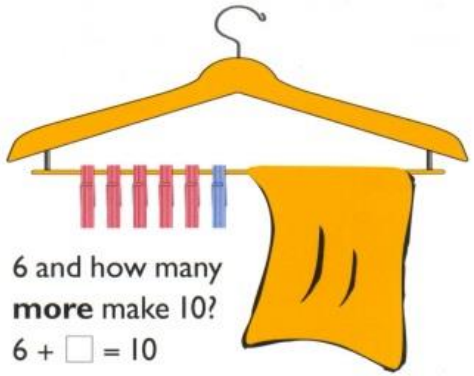


K Rogers

Yr	Addition Strategies	Concrete	Pictorial/ Structural	Abstract
R	Finding the sum of two numbers. Combining 2 parts to make a whole: part part whole model To know the pairs that total 5. ELG: to add two single digit numbers and count on to find the answer	   Use cubes and Numicon to add two numbers together as a group or in a bar.   4 + 2 = 6	  Use pictures to add two numbers together as a group or in a bar.   2 + 4 = 6	Children will annotate enactive and iconic with numerals as they develop this skill. Record addition (as combining 2 or more sets) in pictures. 

Subject Lead: Kimberley Rogers

1	Using a range of manipulatives to add one and two digit numbers to 50.	 Using Numicon to investigate the creation of numbers to 10 and above. First steps to bridging.  6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	e.g. 8 + 5 = 13 13 + 6 = 19  **22 = 8 + 14 (crossing tens boundary)** Start at the larger number on the number line and count on in ones or in one jump to find the answer. Use pictures or a number line. Regroup or partition the smaller number to make 10.  Use the bar model to represent addition:  6 + 4 = 10 4 + 6 = 10	**5 + 12 = 17** Place the larger number in your head and count on the smaller number to find the answer. Number problems like: 7 = ☐ - 9 **7 + 4 =11** If I am at seven, how many more do I need to make 10. How many more do I add on now?  Adding three numbers: **5 + 6 + 2 = 13** **5 + 2 + 6 = ☐**

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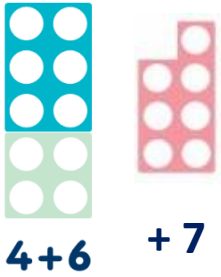
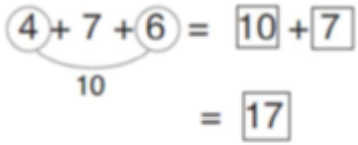
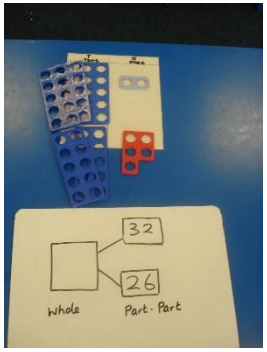
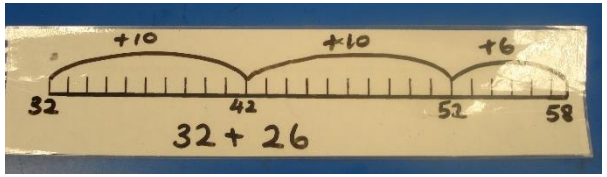
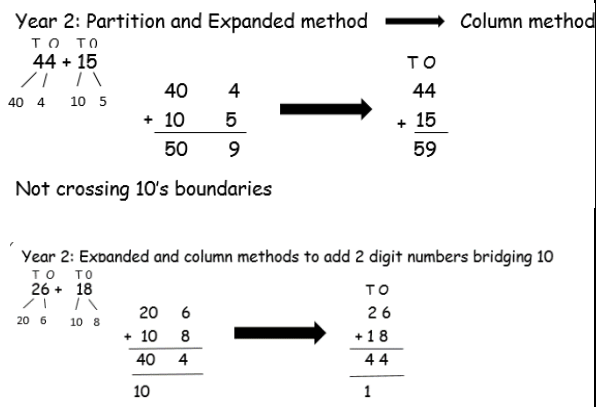
2	Adding three single digits	4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10. Symbols to represent missing numbers eg. □ + △ = 20	 Combine the two numbers that make 10 and then add on the remainder.
2	Adding two 2 digit numbers up to 100	 Use manipulatives to secure understanding of crossing 10's boundaries. 24 + 15 = Add together the ones first then add the tens. Use Cuisenaire first before moving onto place value counters	 Leading to 48 + 36 = 84  or:  Partitioning and recombine using arrow cards.	 Calculate the value of an unknown in a number sentence 30 - □ = 42

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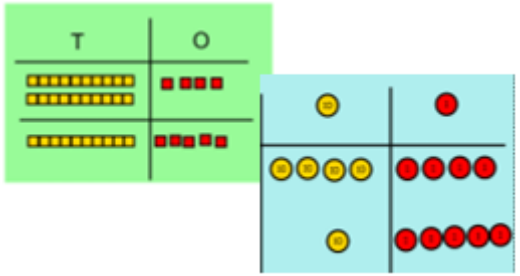
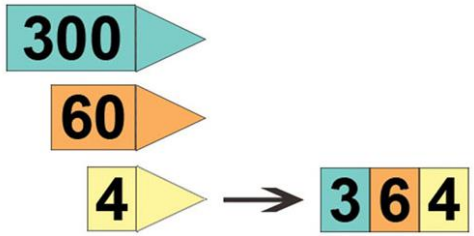
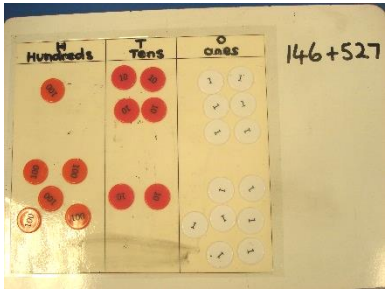
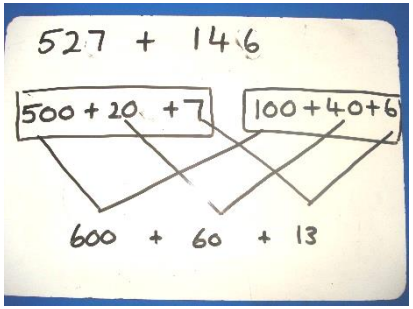
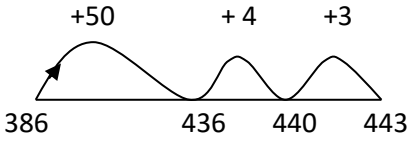
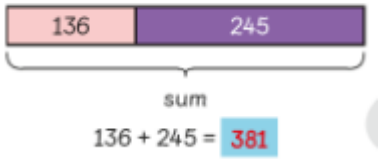
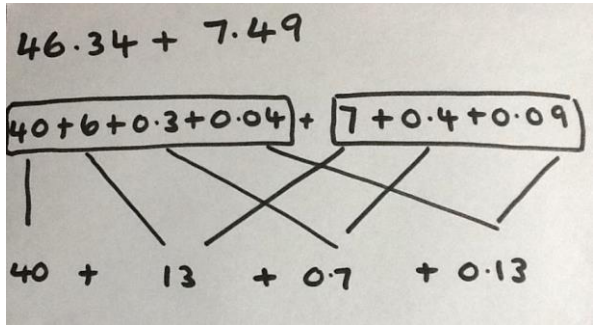
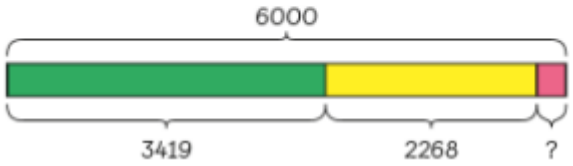
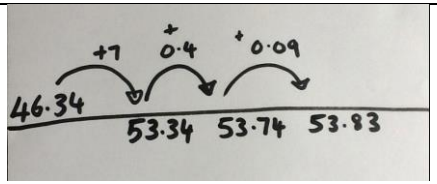
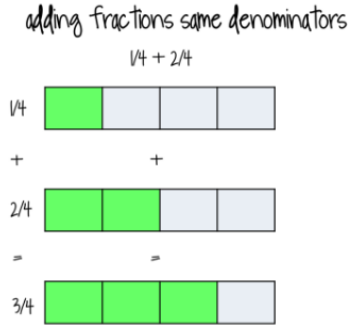
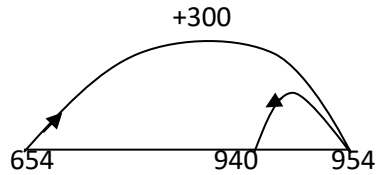
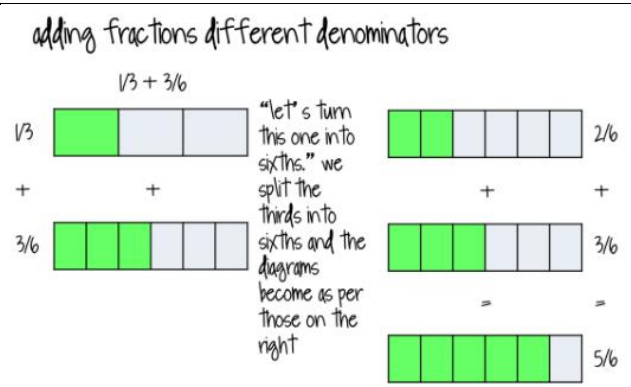
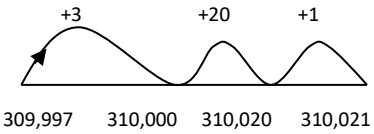
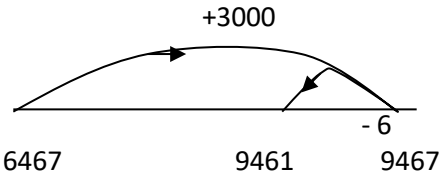
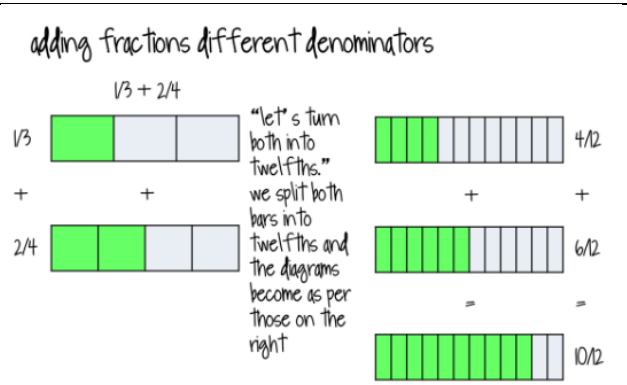
		 Develop to include regrouping.		Partition numbers into T U, developing to H T U (support with arrow cards) eg. 379 = 300 + 70 + 9 Adding TU + TU (including crossing ten) HTU + U (ready to cross tens boundary)
3	Column method including regrouping up 3 digit numbers including tenths	 Begin in the ones column. For every 10 created exchange for a 10 counter. As children move on to decimals and money, decimal place value counters can be used to support learning.	 This informal representation is used to clarify understanding and can be used alongside number lines. counting on in multiples of 100, 10, 1 386 + 57 → 386 + 50 + 7 → 436 + 7 = 443  It will also aid fluency in mental calculations.	Expanded and column methods to add up to 3 digit numbers bridging 10 and 100. Leading to more formal written method: 267 + 85 200 + 60 + 7 + 80 + 5 200 + 140+ 12 H T O 2 5 6 + 1 3 7 3 9 3 1 T O . 1/10 5 8 . 4 + 2 4 . 3 8 2. 7 1 Children must always show place value headings.

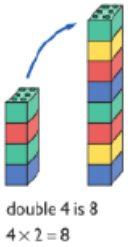
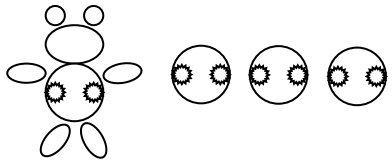
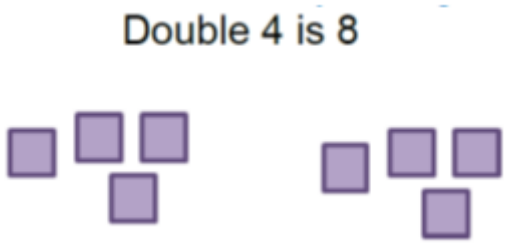
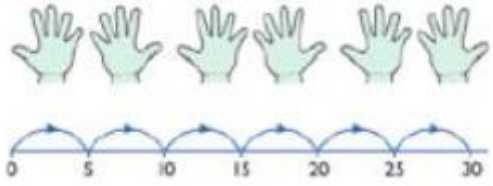
			The bar model reinforces the concept of part part whole. Find the sum of 136 and 245.  	
4	Column method increasing in place value.	Recap on use of place value counters to show exchange.	Expanded method using decimals.  There were 6000 books for sale at a book fair. 3419 books were sold on the first day of the fair and 2268 books were sold on the second day. How many books were left at the end of the second day?  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. $\begin{array}{rcl} 20 & + & 5 \\ \underline{40} & + & \underline{8} \\ 60 & + & 13 \end{array} = 73$ As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\ 1 \end{array}$ Use inverse to check answers.

			 To promote fluency number lines can be used for addition of decimals	
5			Add numbers mentally up to 1000 and beyond (12462 - 2300 =) Use pencil and paper methods to support, record or explain calculations, achieving consistent accuracy. Discuss, explain and compare methods. HTU + HTU, then ThHTU + ThHTU (and 2dp in context of money and measures) A: Compensation (add too much, take off) 654 + 286  Develop expanded horizontal method (least significant numbers first) 7587 + 675 7000 + 500 + 80 + 7 600 + 70 + 5 7000 + 1100 + 150 + 12	Expanded vertical method adding the least significant digits first 7587 + 675 12 (add mentally from top) 150 1100 7000 8262 Standard written methods Develop an efficient standard method that can be applied generally. For example: 47 + 76 = 123 (11) 258 + 87 = 345 (11) 366 + 458 = 824 (11) .

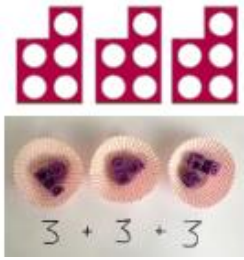
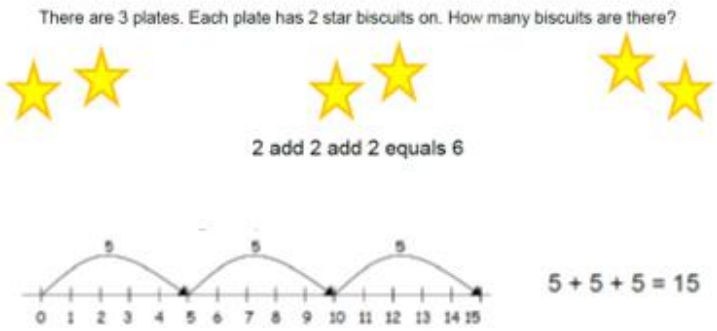
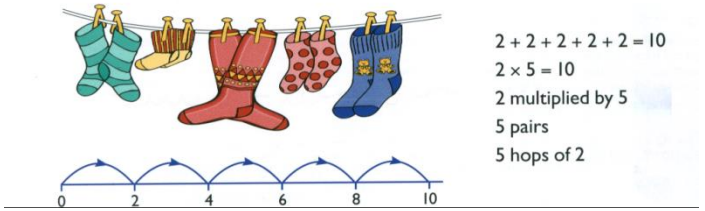
				72.8 +54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 Explain 'carrying' with decimals.
6			Blank number line to support with larger numbers 309,997+24  A. Maintain compensation (add too much, take off) and adjusting 6467 + 2994 	Standard written methods Using 'carrying' 6584 + 5848 12432 111 Extend method to numbers with any number of digits. 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2 Add several numbers with different numbers of digits. For example, find the total of:

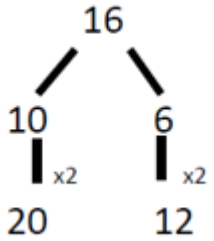
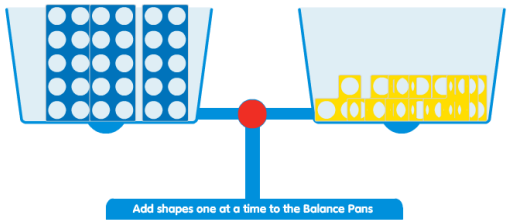
				42, 6432, 786, 3, 4681 Add fractions with different denominations and mixed numbers, using the concept of equivalent fractions. Extend to decimals, add two or more decimal fractions with up to four digits and either one of two decimal places. 124.9 + 7.25 401.2 + 26.85 + 0.71

Treloweth School Agreed Approach to Calculations **2018/2019**

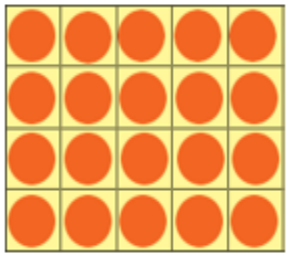
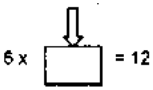
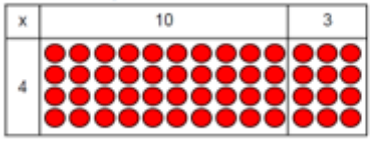
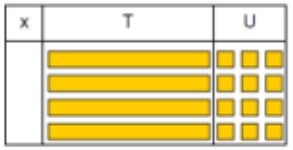
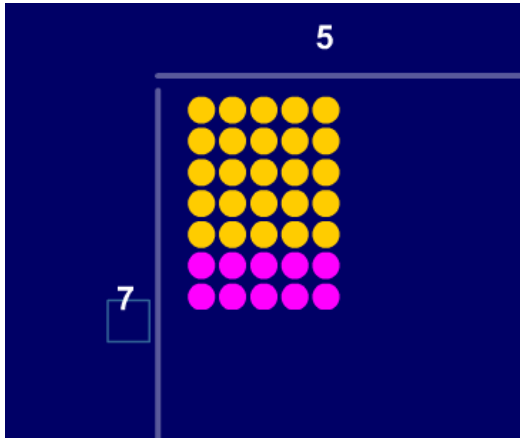
Yr	Multiplication Strategies	Enactive	Iconic	Symbolic
R Tables: 2 and 10	Doubling and finding lots of ELG: solve problems involving doubling and halving.	Use practical activities to show how to double a number.  Real life problems: One teddy has 2 buttons, how many buttons will we need for 3 teddies? 	Draw pictures to show how to double a number.  	Count in multiples of a number aloud using actions.
1 Tables: 2 5 10	Counting in Multiples	  Count in multiples supported by concrete	 Use a number line or pictures to continue support in counting	Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

Subject Lead: Kimberley Rogers

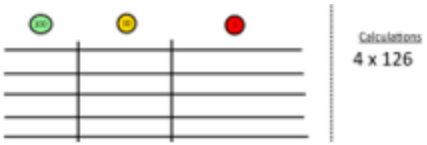
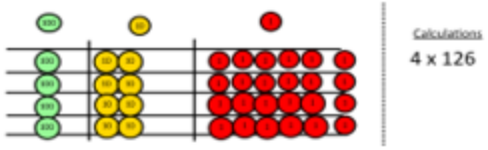
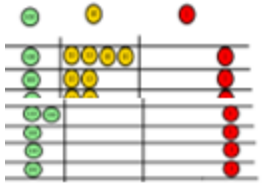
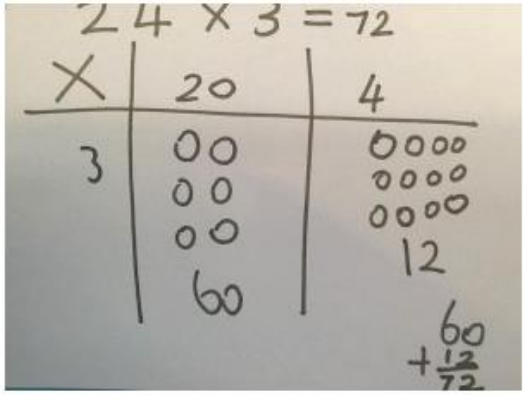
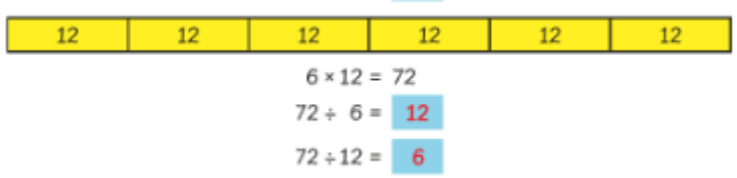
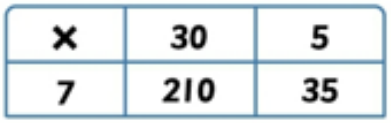
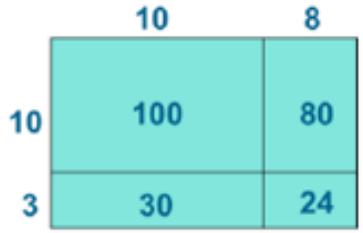
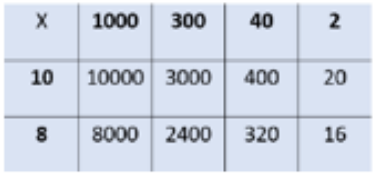
		objects in equal groups.	in multiples.	Write addition sentences to describe objects and pictures. 
1	Repeated Addition	  Use different objects to add equal groups Using multiink, cuisinere, number tracks and bead strings: 	 Explore language of multiplication:  	Record in number sentences: 5 + 5 + 5 + 5 = 20 2 + 2 + 2 + 2 = 8

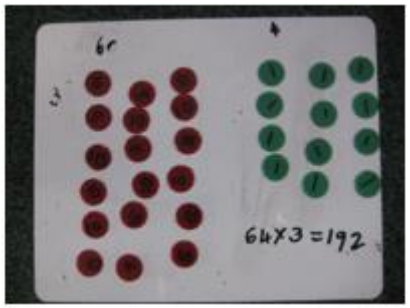
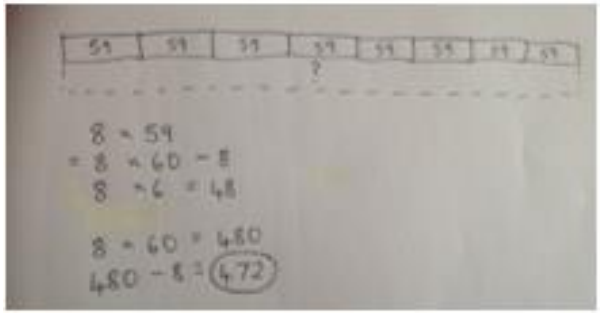
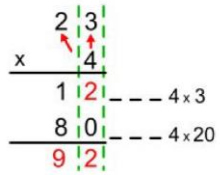
1/2	Partitioning to double (pre grid)	Use objects to model doubling ten and ones practically.	Use Numicon and place value cards to show : 10 x 2 = 4 x 2 = 	Partition a number and then double each part before recombining it back together. 
2	Understanding multiplication	Repeated addition	Use Numicon interactive software to model number sentences linking repeated addition and multiplication. 10 +10+10 = 10 x 3 	**4 × 5 = 10 ☐ 10** **6 ☐ 5 = 15 + 15**

Subject Lead: Kimberley Rogers

2/3	Arrays – showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8  Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15 Empty box sentences using known facts: 
3	Arrays leading to grid	Show link with arrays to first introduce the grid method. Use counters to show arrays:  4 rows of 10 4 rows of 3 Move on to using Deines to move towards a more compact method  4 rows of 13	ITP array to support introduction of grid TU x U 	Introduce grid method TU x U using multiplication tables that they know: x 20 8 4 80 32 → 80 + 32 = 114 Or broken down further: X 10 10 8 4

Subject Lead: Kimberley Rogers

	Grid Method	Use place value counters to show how we are finding groups of a numbers. We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  Then you have your answer.	Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.  Use the bar model  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  **210 + 35 = 245** Moving forward, multiply by a 2 digit number, showing the different rows within the grid method.  

4	Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answers followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  Develop fluency through variation: 2 × 3 = 6 × 7 = 9 × 8 = 2 × 30 = 6 × 70 = 9 × 80 = 2 × 300 = 6 × 700 = 9 × 800 = 20 × 3 = 60 × 7 = 90 × 8 = 200 × 3 = 600 × 7 = 900 × 8 =	Start with long multiplication, reminding children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer 30 + 8 × 7 = 210 + 56 = 266 30 × 7 = 210 8 × 7 = 56 38 × 7 = 210 + 56 = 266 23 x 4 = ?  Moving on to short multiplication (to include TU x U and HTU x U: 38 × 7 = 266

<table>
<tr>
<td>5</td>
<td>Grid method</td>
<td>Place value counters to show multiplication.</td>
<td>TU x TU and HTU x U using grid. Recap using ITP to model TU x TU if necessary.

<table>
<tr><td>×</td><td>20</td><td>7</td><td></td></tr>
<tr><td>50</td><td>1000</td><td>350</td><td>1350</td></tr>
<tr><td>6</td><td>120</td><td>42</td><td>162</td></tr>
<tr><td></td><td></td><td></td><td>1512</td></tr>
</table>
1</td>
<td>T O
3 2
X 2 4
6 0 0 (20 x 30)
4 0 (20 x 2)
1 2 0 (4 x 30)
8 (4 x 2)
7 6 8

Extend to decimals with up to two decimal places. Multiply by a single digit, approximating first. Know that decimal points should line up under each other.</td>
</tr>
<tr>
<td>6</td>
<td colspan="4">Compact method.

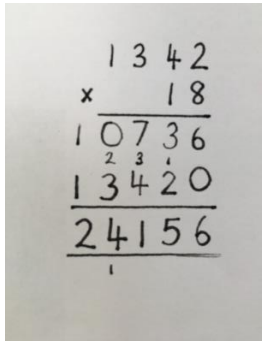
Short multiplication

24 × 6 becomes
2 4
× 6
1 4 4
2
Answer: 144

342 × 7 becomes
3 4 2
× 7
2 3 9 4
2 1
Answer: 2394

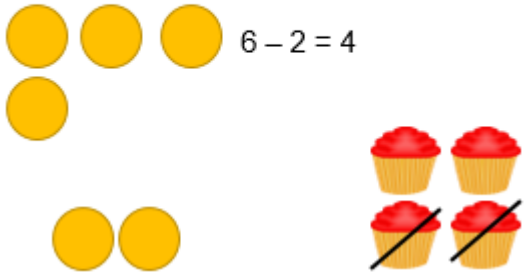
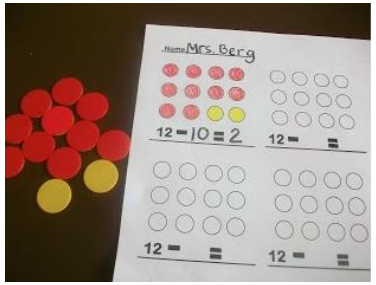
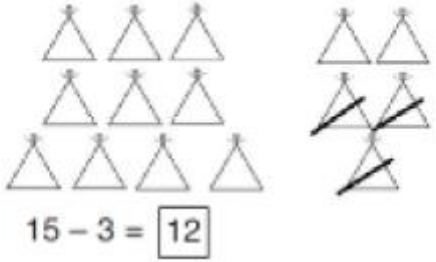
2741 × 6 becomes
2 7 4 1
× 6
1 6 4 4 6
4 2
Answer: 16 446

Leading to:

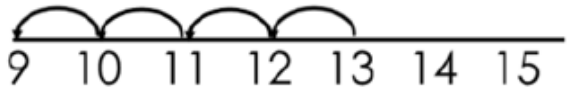
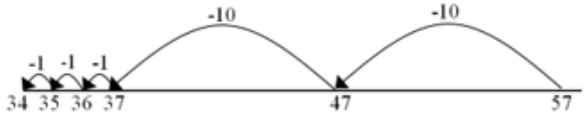
 
1342
× 18
10736
13420
24156</td>
</tr>
</table>

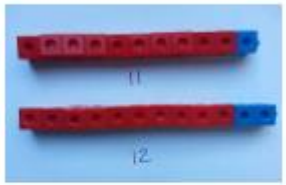
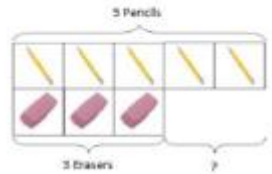
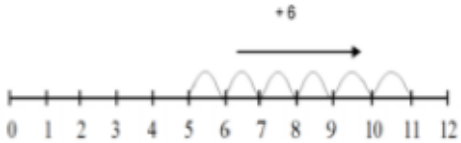
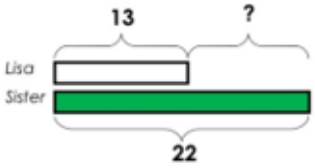
Subject Lead: Kimberley Rogers

Treloweth School Agreed Approach to Calculations **2018/2019**

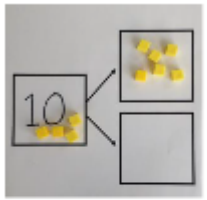
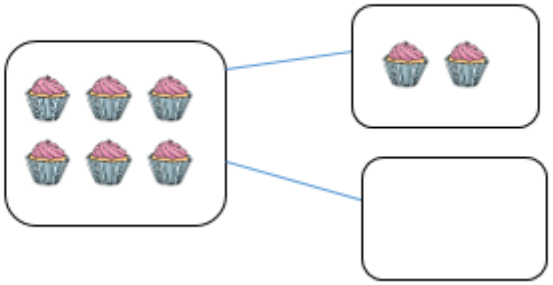
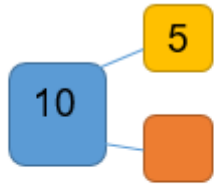
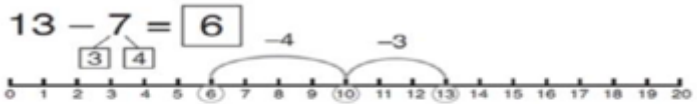
Yr	Subtraction Strategies	Concrete	Pictorial/Structural	Abstract
R /Yr1	Taking away ones ELG: to know 1 more/ 1 less. Subtract 2 single digit numbers and count back to find the answer. Broadbent: count back on a number line to 10 to take away a number.	Use physical objects, counters, cubes etc to show how objects can be taken away.  Using double sided counters 	Cross out drawn objects to show what has been taken away. 	18 -3= 15 8 – 2 = 6

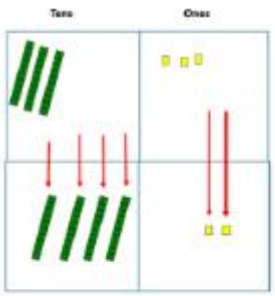
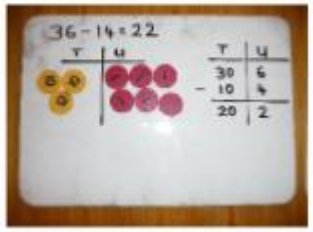
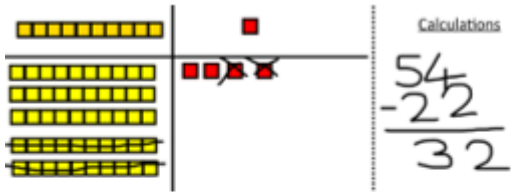
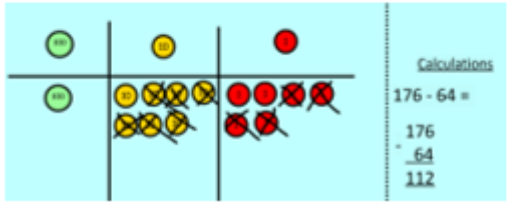
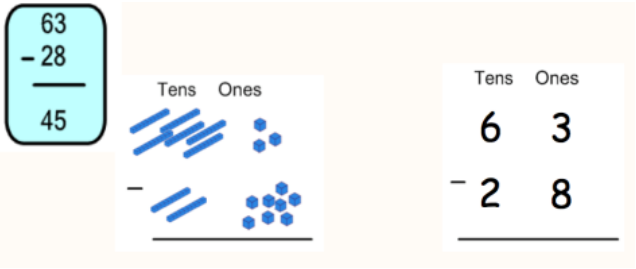
Subject Lead: Kimberley Rogers

1/2	Counting Back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers. Use partitioning to cross boundaries: 15- 7 =8 	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Explore empty boxes by counting back. 13 - ☐ = 9

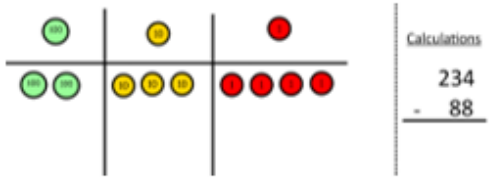
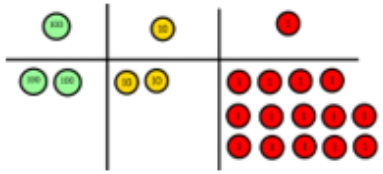
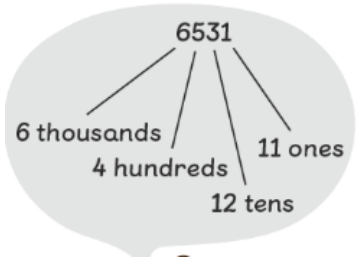
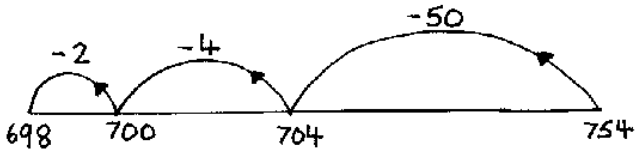
1/2	Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference Use Numicon to find the difference 	 Count on to find the difference. Draw bars to find the difference between 2 numbers. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches; Helen has 15 sandwiches. Find the difference between the number of sandwiches.

Subject Lead: Kimberley Rogers

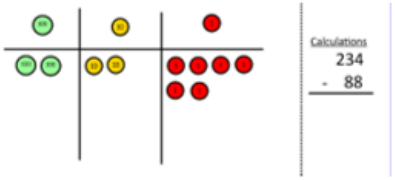
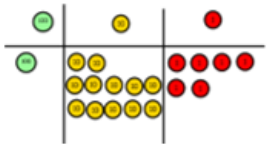
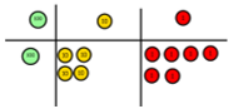
1/2	Part/Part Whole Model	 Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the pa part whole model. 	 Move to using numbers within the part whole model.
1/2	Making 10	14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9	 Start at 13. Take away 3 to reach 10. Then take away t remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

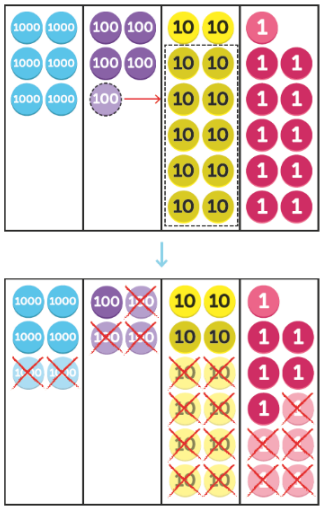
2/3	Column Method Without grouping	 Use Cuisenaire rods to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first. 	 Draw the cuisenaire or place value counters alongside the written calculation to help to show working.  Using base 10: 	TO TO 47 - 23 T O 40 7 - 20 3 20 4 T O 47 - 23 24 H T O 100 70 6 - 60 4 100 10 2 H T O 1 7 6 - 6 4 1 1 2

Subject Lead: Kimberley Rogers

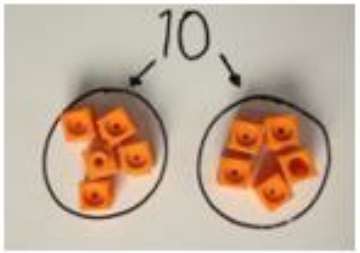
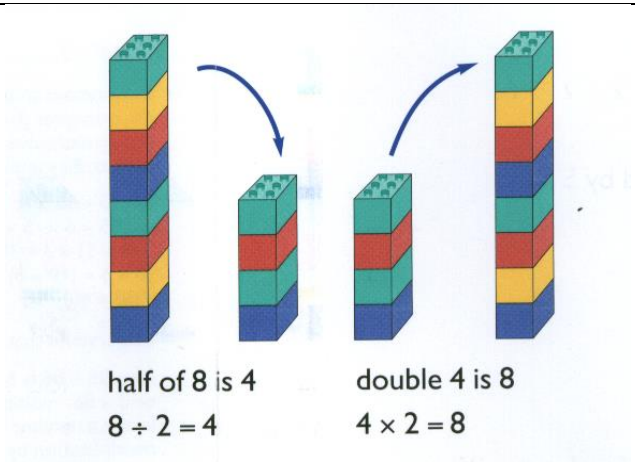
3/4	Column Method with grouping	Use cuisenaire to start with before moving on to place value counters. | Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters  Calculations 234 - 88 Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88	(a) 34 - 8 = 26 20 14 6 4 12 11 ~~5~~ ~~3~~ ~~1~~  Number line subtraction counting back:  	70 + 4 - 20 + 7 ~~70~~ 60 + ~~4~~ 14 - 20 + 7 40 + 7 ~~7~~ 6 ~~4~~ 14 - 2 7 4 7 **Example: 741 - 367** 700 + 40 + 1 - 300 + 60 + 7 ~~700~~ 600 + ~~40~~ 130 + ~~1~~ 11 - 300 + 60 + 7 300 + 70 + 4 ~~7~~ 6 ~~4~~ 13 ~~1~~ 11 - 3 6 7 3 7 4 Th H T O 4 12 11 6 ~~5~~ ~~3~~ ~~1~~ - 2 3 8 5 4 1 4 6

Subject Lead: Kimberley Rogers

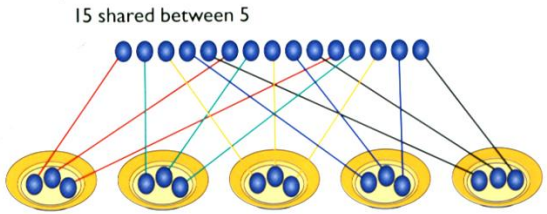
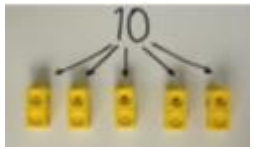
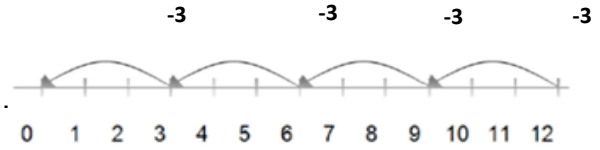
		Now I can subtract my ones.  Calculations 234 - 88 Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Calculations 234 - 88 Now I can take away eight tens and complete my subtraction  Calculations ~~2~~1~~3~~4 - 88 146 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

5/6				$\begin{array}{r} {\scriptstyle 400}\ {\scriptstyle 90}\ {\scriptstyle 13} \\ \not{5}\not{0}\not{3} \\ -\ 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$ $\begin{array}{r} {\scriptstyle 4}\ {\scriptstyle 9}\ {\scriptstyle 13} \\ \not{5}\not{0}\not{3} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$ $\begin{array}{r} {\scriptstyle 3}\ {\scriptstyle 1} \\ 2\not{4}56 - \\ 1385 \\ \hline 1071 \end{array}$ including to 2dp Decimal subtraction: (keeping decimal) £8.42 - £5.78 82.5 km - 3.6 km

Treloweth School Agreed Approach to Calculations **2018/2019**

Yr	Division Strategies	Concrete	Pictorial/ Structural	Abstract
R /1	Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups? 		Share 9 buns between three people. $9 \div 3 = 3$
R/ 1	ELG: to solve problems using doubling, halving and sharing.	Children understand sharing and halving in <u>equal</u> groups, practically and in pictures.		Children can record and begin to explain link: $8 \div 2 = 4$ $4 \times 2 = 8$

Subject Lead: Kimberley Rogers

1 2	Division as sharing	Practical division (as grouping) of buttons, wheels etc. to see how many, for example cars, can be constructed. Focusing on equal groups/lots of 4 wheels per car	Children become confident with the image of sharing: 	Children explain there are 15 eggs shared with 5 people. 15 ÷ 5 = 3
1 2	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects, Numicon, Cuisenaire, place value counters or bead strings to aid understanding.   20 ÷ 5 = 4 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

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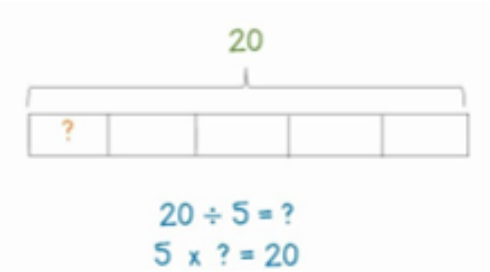
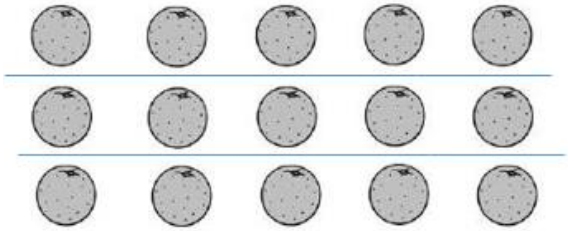
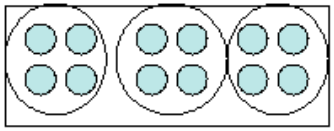
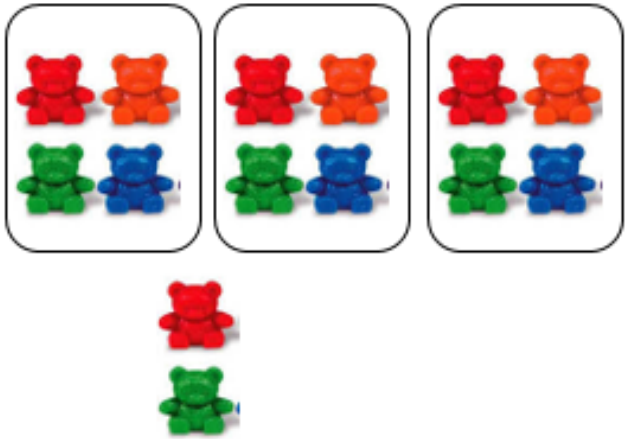
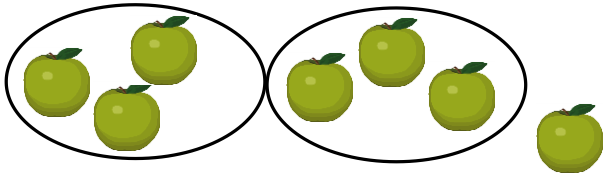
				
2	Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
3		Use tens and ones counters to show division		Children become fluent with representing division stories and images with number sentences they can explain: 96 ÷ 3 = 32

		96 ÷ 3 = 32 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	
3	Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.   Jump back on the number line and show remainders.	Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 ↑ dividend ↑ divisor ↑ quotient ↑ remainder

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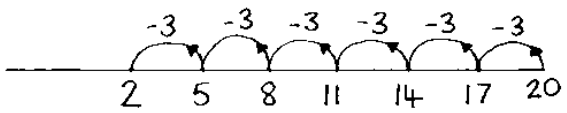
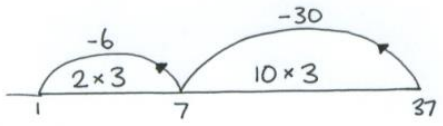
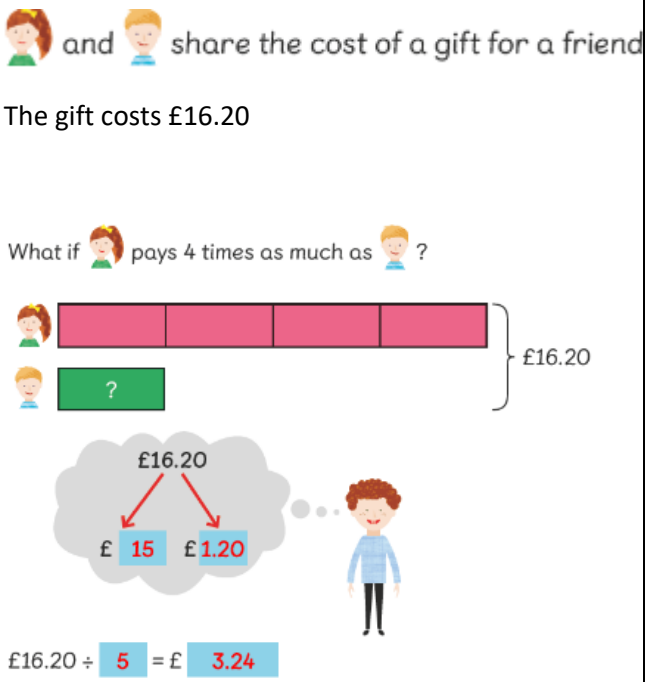
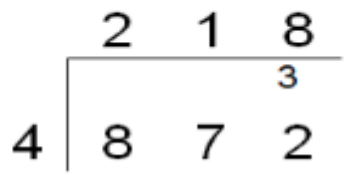
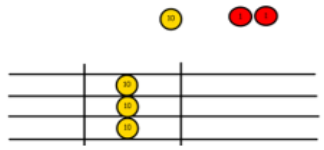
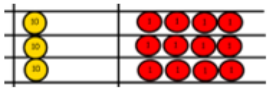
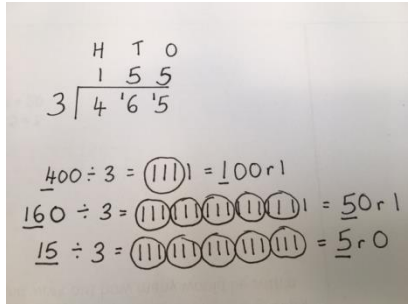
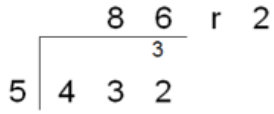
			 Progress to chunking to support understanding: 	
4	Short Division			Ensure children understand that we are finding 'how many lots of the divisor' and can explain: 6)196 − 60 6 ×10 136 − 60 6 ×10 76 − 60 6 ×10 16 − 12 6 × 2 4 32 Answer: 32 R 4 Introduce short division (bus stop method):

				Begin with divisions that divide equally with no remainder. 
4 and 5		42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Use lines to show how many times the divisor can be divided into each value. Alongside the bus stop method. 	Move onto divisions with a remainder  Represent remainder as a fraction 86 2/5 And then as a decimal 86. 4
6	Accuracy and long division	Use Place Value counters to support where necessary.	Finally move into decimal places to divide the total accurately.	

			1 4 . 6 16 21 3 5 | 5 1 1 . 0 Children apply their knowledge of fractions and decimals to solve problems. 432 ÷ 15 becomes 2 8 r 12 1 5 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 Answer: 28 remainder 12 432 ÷ 15 becomes 2 8 1 5 | 4 3 2 3 0 0 15×20 1 3 2 1 2 0 15×8 1 2 $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ 432 ÷ 15 becomes 2 8 · 8 1 5 | 4 3 2 · 0 3 0 ↓ 1 3 2 1 2 0 ↓ 1 2 0 1 2 0 0 Answer: 28·8